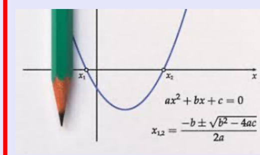
Math 125

Spring 2022

Lecture 16



Class QZ 14

Solve by $\boxed{\text{matrix Method}}$:

$$\begin{cases} x - 2y = 5 \\ 2x + 3y = -4 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 2 & 3 & -4 \end{array}\right] ✓$$

$(-2)R_1 + R_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 7 & -14 \end{array}\right] ✓$$

$R_2 \div 7 \to R_2$ ✓

$$✓\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right] \begin{array}{l} \Rightarrow x = 1 \\ \Rightarrow y = -2 \end{array}$$

✓ $\Rightarrow \boxed{(1,-2)}$

$(2)R_2 + R_1 \to R_1$ ✓

$\{(1,-2)\}$

Introduction to radicals and rational exponents:

Index $\to \sqrt[n]{}$ Radical, Radicand

Ex: $\sqrt[3]{2x-1}$ Index $=3$
Radicand $=2x-1$

$\sqrt[4]{x^2+5x-4}$ Index $=4$
Radicand $=x^2+5x-4$

Whenever index is missing $\Rightarrow$ we assume index $=2$.

$\sqrt{x+8}$ No index $\Rightarrow$ index $=2$
Radicand $=x+8$

when $n=2 \Rightarrow$ Square root
when $n=3 \Rightarrow$ Cube root

For any other $n \Rightarrow$ nth root

$\sqrt[5]{}$ $\Rightarrow$ 5th root

So what does all of these mean?

$\sqrt[n]{\text{Radicand}} = \text{Answer} \iff \text{Answer}^n = \text{Radicand}$

$\sqrt[3]{x} = 2 \iff 2^3 = x$

$\sqrt[5]{x-1} = -3 \iff (-3)^5 = x-1$

$\sqrt{x+2} = 3x-1 \iff (3x-1)^2 = x+2$

No index $\Rightarrow n=2$

Some Restrictions on $\sqrt[n]{\text{Radicand}} = \text{Answer}$:

If n is even,

Radicand ≥ 0 **AND** Answer ≥ 0

If n is odd,

Radicand **AND** Answer must have Same Sign.

Both positive OR Both Negative

Ex: $\sqrt[4]{81} = 3$ why? $3^4 = 81$

even index

Both positive

$\sqrt[5]{-32} = -2$ why? $(-2)^5 = -32$

odd index

Both negative

$\sqrt[3]{27} = 3$ why? $3^3 = 27$

Both positive

$\sqrt[6]{-10}$ undefined why? even index but radicand < 0

No index $\Rightarrow n = 2$

$\sqrt{100} = -10$ false why? even index but answer < 0

Find the domain for $f(x) = \sqrt{x-4}$

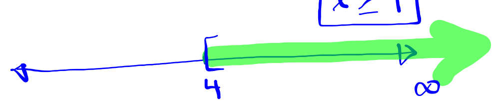
$f(x) = \sqrt{x-4}$

No index $\Rightarrow$ index $= 2 \Rightarrow \underbrace{\text{even index}}$

Radicand ≥ 0

$x - 4 \geq 0$

$\boxed{x \geq 4}$



Domain: $[4, \infty)$

Find the domain of $g(x) = \sqrt[3]{2x+5}$

odd index

$\Rightarrow$ No restriction on radicand

Domain: $(-\infty, \infty)$

April 12, 2022

Rational Exponents

Fraction

$x^{\frac{2}{3}}$, $y^{\frac{3}{5}}$, $(x-1)^{\frac{1}{2}}$, $(2x+3)^{\frac{5}{4}}$

Exponential Rules:

$x^m \cdot x^n = x^{m+n}$ — $x^5 \cdot x^2 = x^{5+2} = x^7$

$\frac{x^m}{x^n} = x^{m-n}$ — $\frac{x^5}{x^2} = x^{5-2} = x^3$

$(x^m)^n = x^{m \cdot n}$ — $(x^5)^2 = x^{5 \cdot 2} = x^{10}$

Simplify $x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = \boxed{x^{\frac{5}{6}}}$

Simplify $\frac{x^{1/2}}{x^{1/3}} = x^{\frac{1}{2}-\frac{1}{3}} = \boxed{x^{\frac{1}{6}}}$

Simplify $\left[x^{\frac{2}{3}}\right]^{\frac{2}{5}} = x^{\frac{2}{3}\cdot\frac{2}{5}} = \boxed{x^{\frac{4}{15}}}$

Simplify $x^{\frac{2}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{2}{3}+\frac{1}{4}} = \boxed{x^{\frac{11}{12}}}$

$\frac{2 \cdot 4}{3 \cdot 4} + \frac{1 \cdot 3}{4 \cdot 3} = \frac{8}{12} + \frac{3}{12} = \boxed{\frac{11}{12}}$

Simplify $\frac{x^{\frac{3}{5}}}{x^{1/2}} = x^{\frac{3}{5}-\frac{1}{2}} = \boxed{x^{\frac{1}{10}}}$

$\frac{3 \cdot 2}{5 \cdot 2} - \frac{1 \cdot 5}{2 \cdot 5} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

Simplify $\left[x^{\frac{2}{3}}\right]^{\frac{3}{4}} = x^{\frac{\cancel{2}^{1}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{4}_{2}}} = \boxed{x^{\frac{1}{2}}}$

Relationship between radicals and rational exponent:

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$\sqrt[10]{x} = x^{\frac{1}{10}}$

$\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$\sqrt[5]{x^3} = x^{\frac{3}{5}}$

$\sqrt{x} = x^{\frac{1}{2}}$

Simplify: $\sqrt[6]{x^2} = x^{\frac{2}{6}}$

$= x^{\frac{1}{3}}$

$= \sqrt[3]{x^1} = \boxed{\sqrt[3]{x}}$

Simplify: $\sqrt[12]{x^6} = x^{\frac{6}{12}}$

$= x^{\frac{1}{2}}$

$= \sqrt[2]{x^1} = \boxed{\sqrt{x}}$

Final Answer

Simplify $\sqrt[3]{x} \cdot \sqrt{x} = x^{\frac{1}{3}} \cdot x^{\frac{1}{2}}$

$= x^{\frac{1}{3}+\frac{1}{2}}$

$= x^{\frac{5}{6}}$

$= \boxed{\sqrt[6]{x^5}}$

Simplify:

$$\frac{\sqrt[5]{x^3}}{\sqrt[4]{x}} = \frac{x^{\frac{3}{5}}}{x^{\frac{1}{4}}}$$

$$= x^{\frac{3}{5}-\frac{1}{4}}$$

$$= x^{\frac{7}{20}}$$

$$= \boxed{\sqrt[20]{x^7}}$$

$$\frac{3\cdot 4}{5\cdot 4} - \frac{1\cdot 5}{4\cdot 5} =$$

$$\frac{12}{20} - \frac{5}{20} =$$

$$\frac{12-5}{20} = \frac{7}{20}$$

Simplify

$$\sqrt[5]{\sqrt[4]{x^3}} = \sqrt[5]{x^{\frac{3}{4}}}$$

$$= \left[x^{\frac{3}{4}} \right]^{\frac{1}{5}}$$

$$= x^{\frac{3}{4}\cdot\frac{1}{5}} = x^{\frac{3}{20}}$$

$$= \boxed{\sqrt[20]{x^3}}$$

Simplify: $\dfrac{\sqrt[3]{x}\cdot\sqrt[4]{x}}{\sqrt{x}} = \dfrac{x^{\frac{1}{3}}\cdot x^{\frac{1}{4}}}{x^{\frac{1}{2}}}$

No index → index=2

$\dfrac{7}{12}-\dfrac{1\cdot 6}{2\cdot 6}$

$=\dfrac{7}{12}-\dfrac{6}{12}$

$=\dfrac{7-6}{12}=\dfrac{1}{12}$

$$= \frac{x^{\frac{1}{3}+\frac{1}{4}}}{x^{\frac{1}{2}}} = \frac{x^{\frac{7}{12}}}{x^{\frac{1}{2}}} = x^{\frac{7}{12}-\frac{1}{2}}$$

$$= x^{\frac{1}{12}} = \sqrt[12]{x^1}$$

$$\boxed{=\sqrt[12]{x}}$$

SG 11:

Solve $\begin{cases} y = 3x - 9 \\ x^2 = 2y + 10 \end{cases}$

System of non-linear equations

$x^2 = 2(3x-9)+10$

$x^2 = 6x - 18 + 10$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

→ By Zero-Factor Prop.

$x-4=0$ OR $x-2=0$

$\boxed{x=4}$ $\boxed{x=2}$

when $x=4$

$y=3(4)-9=3$

$(4,3)$

when $x=2$

$y=3(2)-9=-3$

$(2,-3)$

Final Ans:

$\{(4,3),(2,-3)\}$

Solve $\begin{cases} \boxed{y} = x^2+1 \\ 4x - \boxed{y} = -1 \end{cases}$

$4x - (x^2+1) = -1$

$4x - x^2 - 1 = -1$

$-x^2 + 4x - 1 + 1 = 0$

$-x^2 + 4x = 0$

Multiply by -1

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ $\quad$ $x - 4 = 0$

$x = 4$

when $x = 0$

$y = 0^2 + 1 = 1 \Rightarrow (0, 1)$

when $x = 4$

$y = 4^2 + 1 = 17 \Rightarrow (4, 17)$

$\{(0,1), (4,17)\}$

Solve by addition Method:

$\begin{cases} 4x^2 + y^2 = 13 \\ x^2 + y^2 = 10 \end{cases}$ $\quad -1$

$\Rightarrow \begin{cases} 4x^2 + \cancel{y^2} = 13 \\ -x^2 - \cancel{y^2} = -10 \end{cases}$

$3x^2 = 3$

$x^2 = 1 \rightarrow \boxed{x = \pm 1}$

$1 + y^2 = 10$

$y^2 = 9 \rightarrow \boxed{y = \pm 3}$

$\{(1,3), (1,-3), (-1,3), (-1,-3)\}$

Solve by addition Method:

$$\begin{matrix} 3 \\ -2 \end{matrix}\begin{cases} 3x^2 + 2y^2 = 35 \\ 4x^2 + 3y^2 = 48 \end{cases} \Rightarrow \begin{cases} 9x^2 + \cancel{6y^2} = 105 \\ -8x^2 - \cancel{6y^2} = -96 \end{cases}$$

$$x^2 = 9$$

$$x = \pm 3$$

$$3(9) + 2y^2 = 35$$

$$27 + 2y^2 = 35$$

$$2y^2 = 35 - 27$$

$$2y^2 = 8$$

$$y^2 = 4 \rightarrow y = \pm 2$$

$$\{(3,2), (3,-2), (-3,2), (-3,-2)\}$$

The difference between squares of two numbers is 5.

$$x^2 - y^2 = 5$$

Twice the square of the second number subtracted $\boxed{\text{from}}$ three times the square of the first number is 19.

$$3x^2 - 2y^2 = 19$$

$$\begin{matrix} -2 \\ \\ \end{matrix}\begin{cases} x^2 - y^2 = 5 \\ 3x^2 - 2y^2 = 19 \end{cases} \Rightarrow \begin{cases} -2x^2 + 2y^2 = -10 \\ 3x^2 - 2y^2 = 19 \end{cases}$$

$$x^2 = 9$$

$$\boxed{x = \pm 3}$$

$$9 - y^2 = 5$$

$$y^2 = 4 \rightarrow \boxed{y = \pm 2}$$

The numbers are 3 & 2, 3 & -2, -3 & 2, and -3 & -2.

Introduction to Variations:

1) Directly

y varies directly as x. $\Rightarrow y = Kx$

2) Inversely

y varies inversely as x. $\Rightarrow y = \frac{K}{x}$

$K \rightarrow$ Constant of Variation

3) Joint

y varies $\boxed{\text{directly}}$ as x^2 $\Rightarrow y = Kx^2$

y is 50 when x is 5.

$50 = K \cdot 5^2$

$50 = K \cdot 25$

$\boxed{K = 2}$

Find y when x is 10.

$y = 2x^2$

$y = 2(10)^2$

$\boxed{y = 200}$

y varies inversely as $\sqrt{x}$. $\Rightarrow y = \frac{K}{\sqrt{x}}$

y is 10 when x is 4.

$10 = \frac{K}{\sqrt{4}}$

$10 = \frac{K}{2}$

$\boxed{K = 20}$

find y when x is 25.

$y = \frac{20}{\sqrt{x}}$

$y = \frac{20}{\sqrt{25}} = \frac{20}{5} = \boxed{4}$

$\boxed{y = 4}$

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y varies directly as x^3. $\Rightarrow y = Kx^3$

y is 24 when x is 2.

$24 = K(2)^3$

$24 = 8K$

$\boxed{K = 3}$

$y = 3x^3$

Find y when x is -5.

$y = 3(-5)^3 = 3(-125)$

$= \boxed{-375}$

y varies $\boxed{\text{inversely}}$ as x^2.

$$y = \frac{K}{x^2}$$

y is 10 when x is 5.

$$10 = \frac{K}{5^2}$$

$$\boxed{K = 250}$$

Find y when x is 10.

$$y = \frac{250}{x^2}$$

$$y = \frac{250}{10^2} = \frac{250}{100} = \boxed{2.5}$$

2.5

Z varies directly as x^2 and inversely as $\sqrt{y}$.

$$Z = \frac{Kx^2}{\sqrt{y}}$$

Z is 50 when x is 5 and y is 4.

$$50 = \frac{K \cdot 5^2}{\sqrt{4}}$$

$$50 = \frac{K \cdot 25}{2}$$

$$100 = 25K$$

$$\boxed{K = 4}$$

$$Z = \frac{4x^2}{\sqrt{y}}$$

Find Z when $x = 6$, and $y = 9$.

$$Z = \frac{4 \cdot 6^2}{\sqrt{9}} = \frac{4 \cdot 36}{3} = \boxed{48}$$

Z varies directly as $\sqrt{x^2+y^2}$. $\Rightarrow Z = K\sqrt{x^2+y^2}$

Z is 50 when $x=3$ & $y=4$. $\Rightarrow 50 = K\sqrt{3^2+4^2}$

$50 = K\sqrt{25}$

Find Z when $x=6$ & $y=8$.

$50 = 5K$

$\boxed{K=10}$

$Z = 10\sqrt{x^2+y^2}$

$Z = 10\sqrt{6^2+8^2} \Rightarrow \boxed{Z = 100}$

$= 10\sqrt{36+64} = 10\sqrt{100} = 10 \cdot 10 = 100$